LAU
الجامعة اللبنانية الأميركية
Lebanese American University

LAU
School of Engineering

Department of Electrical
and Computer Engineering

ELE 201: Electrical Circuits I
Exam 3 – Spring 2014

Duration: **1 hour 20 minutes**
Start Time: 9:30 am

Date: 15/05/2014
Dr. Dani TANNIR

Name: SOLUTION **ID#:** _____

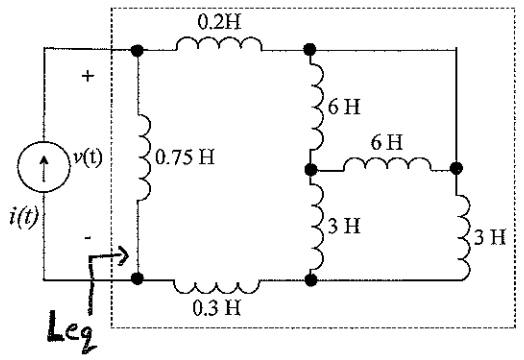
INSTRUCTIONS:

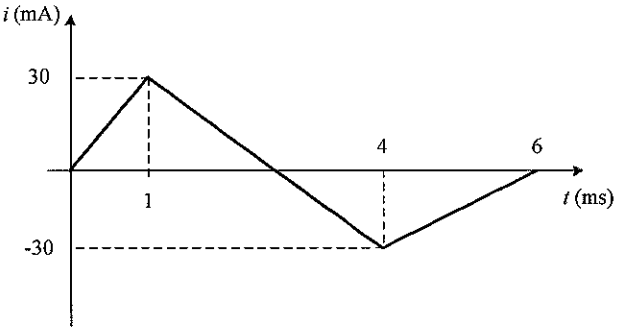
- Answer each of the following questions in the space provided.
- This is a closed-book exam.
- If something is not clear, state your assumptions.
- Programmable calculators are not allowed.
- The number of marks for each question is specified next to it.
- The total number of marks is 100.

1	2	3	4	Total
/25	/25	/25	/25	

Question 1 (25 marks)

The waveform for the current source supplying the following inductor circuit is as shown below





(6) a. Determine the equivalent inductance L_{eq} as seen at the terminals of the current source.
(8) b. Determine the expression for the voltage $v(t)$ across the terminals of the equivalent circuit.
(5) c. Sketch the voltage waveform versus time (label key points).
(6) d. Determine the energy stored in the equivalent inductive circuit at t = 1.5ms and at t = 4.5ms.

a) $L_{eq} = \left[\left[\left[(6\|6)+3\right]\|3\right]+0.2+0.3\right]\|0.75$

$= 2.5\|0.75 = 0.58H$

b) $v(t) = L_{eq}\frac{di}{dt}$

$i(t) = \begin{cases} 30t & 0<t<1ms \\ -20t+50m & 8<t<4ms \\ 15t-90m & 4<t<6ms \end{cases}$

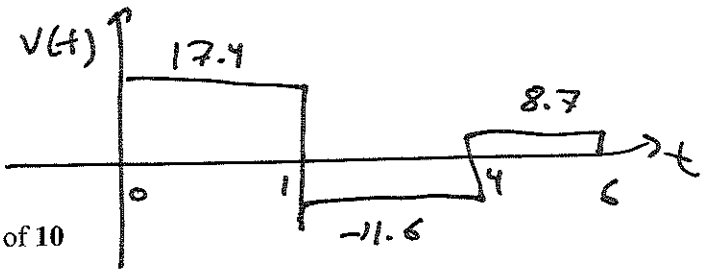
$v(t) = L\frac{di}{dt} = \begin{cases} (0.58)(30) & 0<t<1m \\ (0.58)(-20) & 1<t<4m \\ (0.58)(15) & 4<t<6m \end{cases}$

$v(t) = 17.4$

$= -11.6$

$= 8.7$

c)



d) $E = \frac{1}{2} L i(t)^2$

$i(1.5m) = 20m$

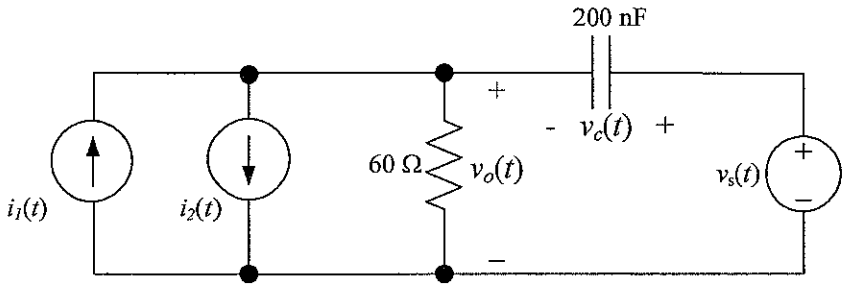
$i(4.5m) = -22.5m$

$E(1.5m) = 0.116mJ$

$E(4.5m) = 0.147mJ$

Question 2 (25 marks)

Consider the following circuit:



a. Calculate $v_o(t)$ in the circuit shown if $i_1(t) = 200\cos(10^5t + 60)$ mA, $i_2(t) = 100\sin(10^5t + 90)$ mA and $v_s(t) = -10\sin(10^5t - 30)$ V.

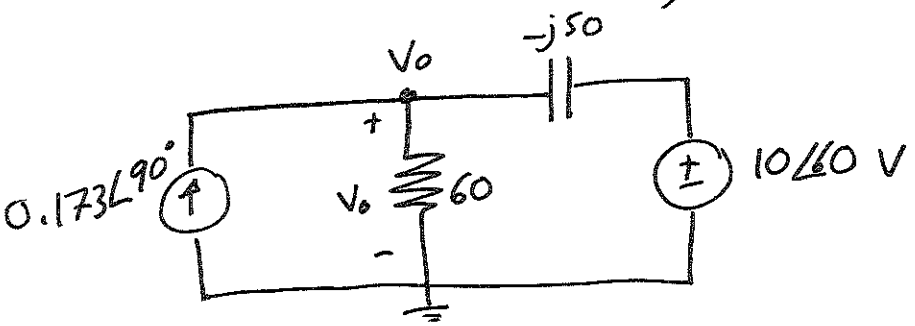
b. By how many degrees does $v_o(t)$ lag $v_s(t)$?

c. Sketch the phasor diagram showing $\mathbf{V}_s$, $\mathbf{V}_o$ and $\mathbf{V}_c$ (label key magnitudes and angles).

a) $i_1(t) = 200\cos(10^5t + 60)m \Rightarrow I_1 = 0.2\angle 60$

$i_2(t) = 100\cos(10^5t)m \Rightarrow I_2 = 0.1\angle 0$

$v_s(t) = -10\cos(10^5t - 30) \Rightarrow V_s = 10\angle 60$



$$-0.173\angle 90^\circ + \frac{V_o}{60} + \frac{V_o - 10\angle 60}{-j50} = 0$$

$$V_o = \frac{0.173\angle 90 + \frac{10\angle 60}{-j50}}{\frac{1}{60} + \frac{1}{-j50}} = \frac{-0.173 + j0.273}{0.0167 + j0.02}$$

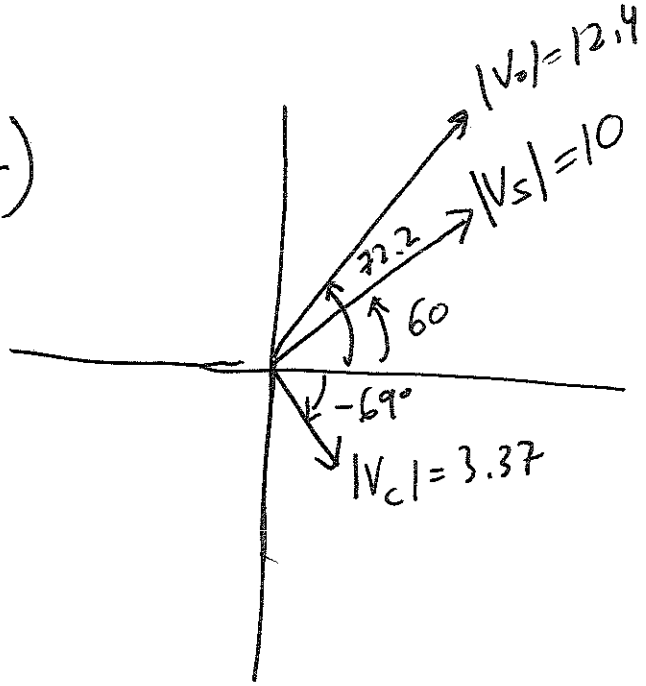
$$= 3.8 + j11.8$$

$$= 12.4\angle 72.2^\circ \text{ V}$$

$\Rightarrow v_o(t) = 12.4\cos(10^5t + 72.2)$

b) $\angle V_s - \angle V_o = 60 - 72.2 = \boxed{-12.2^\circ}$

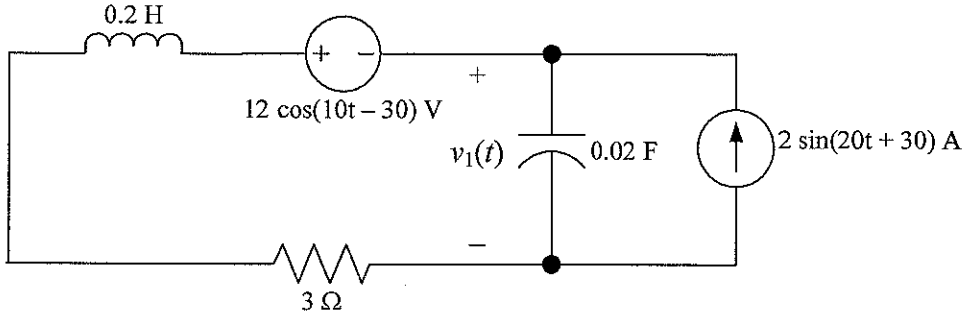
c)



$$V_c = V_s - V_o = 3.37\angle -68.97$$

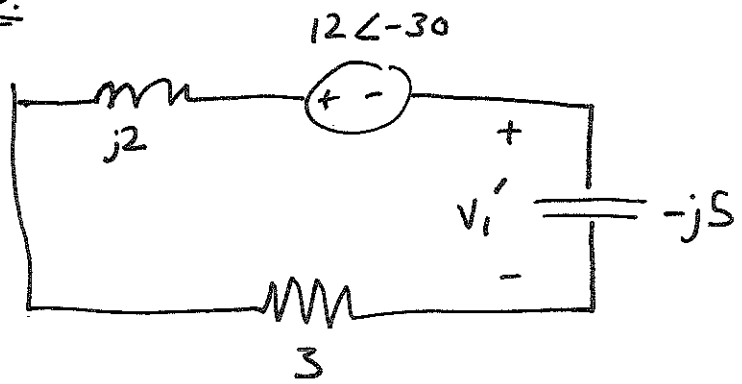
Question 3 (25 marks)

Determine $v_1(t)$ in the following circuit.



Different Frequencies ⇒ Must use superposition!

VS.

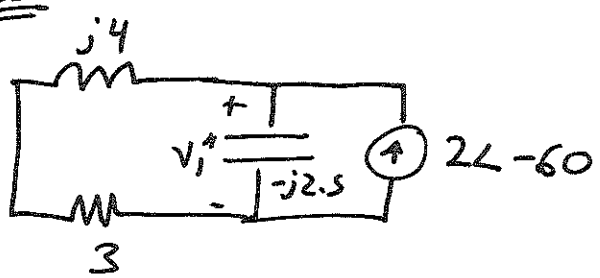


$$V_1' = -\left(\frac{-jS}{3+j2-jS}\right)12\angle -30$$

$$= -3.66 + j13.66$$

$$= 14.14\angle 105\ V$$

CS.



$$V_1'' = \left(\frac{3+j4}{3+j4-j2.5}\ 2\angle -60\right)(-j2.5)$$

$$= (2.488 - j\,1.64)(-j2.5)$$

$$= -4.1068 - j\,6.22$$

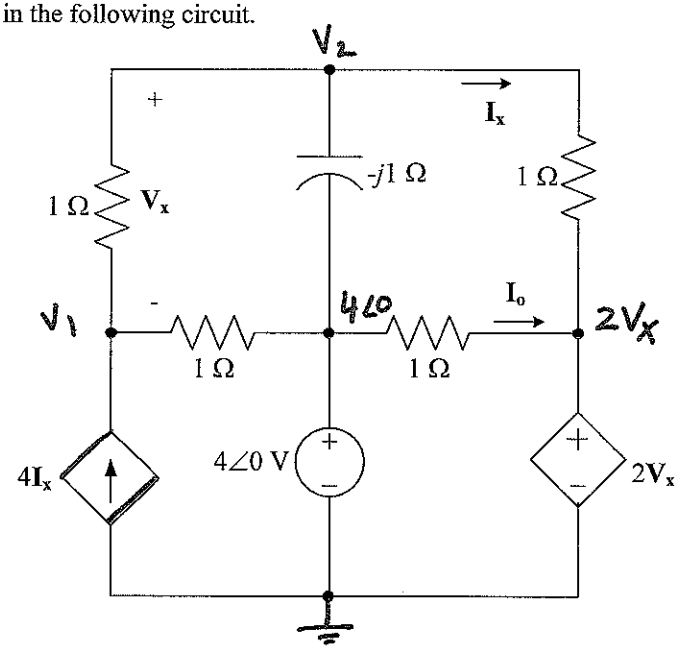
$$= 7.45\angle -123.4$$

[redacted]

$$v_1(t) = 14.14\cos(10^5 t + 105^\circ) + 7.45\cos(20^5 t - 123.4)\ V$$

Question 4 (25 marks)

Find the phasor current I_o in the following circuit.



Node equation at V_1

$$\frac{V_1 - V_2}{1} + \frac{V_1 - 4\angle 0}{1} - 4I_x = 0$$

$$I_x = V_2 - 2V_x = V_2 - 2V_2 + 2V_1 = 2V_1 - V_2$$

$$V_x = V_2 - V_1$$

$$V_1 - V_2 + V_1 - 4\angle 0 - 8V_1 + 4V_2 = 0$$

$$-6V_1 + 3V_2 = 4\angle 0$$

$$V_2 = \frac{4 + 6V_1}{3}$$

Node equation at V_2

$$\frac{V_2 - V_1}{1} + \frac{V_2 - 4\angle 0}{-j1} + \underbrace{\frac{2V_1 - V_2}{I_x}} = 0$$

$$V_2 - V_1 + jV_2 - j4 + 2V_1 - V_2 = 0$$

$$V_1 + jV_2 = j4$$

$$V_1 + \left(\frac{4 + 6V_1}{3}\right)(j) = j4$$

$$V_1 + j\frac{4}{3} + j2V_1 = j4$$

$$V_1(1 + j2) = j\left(\frac{8}{3}\right)$$

$$I_o = \frac{4 - 2V_x}{1}$$

$1.33\angle -126.8$

$-0.8 - j1.067$

$V_1 = 1.067 + j0.533$

$1.2\angle 26.6$

$V_2 = 3.467 + j1.067$

$V_x = 2.4 + j0.533$